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Nonlocal charges of T-dual strings

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Abstract: We obtain sets of infinite number of conserved nonlocal charges of strings in a flat space and pp-wave backgrounds, and compare them before and after T-duality transformation. In the flat background the set of nonlocal charges is the same before and after the T-duality transformation with interchanging odd and even-order charges. In the IIB pp-wave background an infinite number of nonlocal charges are independent, contrast to that in a flat background only the zero-th and first order charges are independent. In the IIA pp-wave background, which is the T-dualized compactified IIB pp-wave background, the zero-th order charges are included as a part of the set of nonlocal charges in the IIB background. To make this correspondence complete a variable conjugate to the winding number is introduced as a Lagrange multiplier in the IIB action à la Buscher's transformation.

Keywords: Integrable Hierarchies, String Duality, AdS-CFT Correspondence, Global Symmetries.

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Contents

1. Introduction

Integrability is one of the most important guiding principles to find correspondences beyond the BMN limit [1] of AdS/CFT correspondence. After generalizations of BMN's work [2, 3] the Bethe ansatz was brought to use in this problem [4], and integrable spin chain models have been shown to describe semiclassical spinning strings [5] as well as superconformal Yang-Mills theories [6]. Those techniques are used to examine the correspondence between the string energy spectrum and the dilatation operator [7, 8] (see [9] for review). For an integrable system conserved quantities are not only the above energy operator but also an infinite set of conserved nonlocal charges [10]. The existence of an infinite set of conserved nonlocal charges implies the absence of particle production and the factorization of the S-matrix [10, 11].

Brezin, Itzykson, Zinn-Justin and Zuber (BIZZ) gave a simple derivation of these nonlocal charges [12]: Assume that one has a set of conserved and "flat" currents J_α, $\partial_\alpha J^\alpha = 0$ and $\epsilon^{\alpha\beta}(\partial_\alpha J_\beta - J_\alpha J_\beta) = 0$. Then an infinite set of nonlocal currents $\mathcal{J}_{[N]\alpha}$ are constructed inductively. The zero-th order current is set to be the Noether current $\mathcal{J}_{[0]\alpha} = J_\alpha$. The first order current, $\mathcal{J}_{[1]\alpha}$, is determined by the "flatness" condition as $\epsilon^{\alpha\beta}(\partial_\alpha J_\beta - J_\alpha J_\beta) = \partial_\alpha \mathcal{J}^\alpha_{[1]} = 0$ where the nonabelian structure of J_α gives rise to the nonlocality of $\mathcal{J}_{[1]\alpha}$. The "flatness" condition is a "dual" equation of the two-dimensional

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worldsheet which is closely related to the Buscher T-duality [13]. In this paper we examine whether the two-dimensional dual formulation required in the nonlocal charges shares any concept with the two-dimensional transformation of T-duality.

Bena, Roiban and Polchinski showed the existence of an infinite set of conserved nonlocal charges of a superstring in the $AdS_5 \times S^5$ background [14], while the one for the bosonic part was shown in [15] and the one in manifestly κ-invariant way was shown in [16]. Additional discussion of nonlocal charges of strings in the AdS background can be found in refs. [17 – 24]. For an integrable model there exist a spectral parameter which relates an infinite set of "local" charges and an infinite set of nonlocal charges. Recently it has been shown that an infinite number of conserved "local" charges for a superstring in the $AdS_5 \times S^5$ background are combinations of the Virasoro constraint and the κ symmetry constraints [25] which govern the target space field equations. So it is natural to expect that the nonlocal charges reflect the spacetime isometry and stringy symmetry of the target space. In order to figure out physical meanings of nonlocal charges we compute nonlocal charges explicitly. In this paper a flat space background and the IIB and IIA pp-wave backgrounds are examined so the nonlocal integration can be performed in terms of oscillators.

The existence of an infinite set of nonlocal charges in the pp-wave background was shown by Alday [26]. We will further develop the work by (1) computing higher order nonlocal charges with respect to complete set of isometry generators, and (2) computing nonlocal charges of a string in the pp-wave background before and after T-dual transformation. The IIB pp-wave background is compactified and T-duality transformation is performed as [27], then the string in the IIA pp-wave background is quantized [28, 29]. For the IIA background we will focus on the Noether charges which are the zero-th order charges, and we will examine whether they are related to the nonlocal charges of the IIB background.

We also clarify the procedure of constructing nonlocal charges of strings in a flat and the pp-wave background which are described by the inhomogeneous $O(n)$ nonlinear sigma model. The BIZZ procedure [12] was originally applied to a homogeneous $O(n)$ non-linear sigma model; the nonlocal charges are systematically constructed from a conserved and "flat" current. This current is a $O(n)$ left-invariant current and satisfies the Maurer-Cartan (M.C.) equation for the $o(n)$ algebra which is the "flatness" condition. For a string in a flat background the geometry may be regarded as a coset sigma model of the Poincaré group over the Lorentz group and the string coordinate is the coset parameter. But the left-invariant current $J_\alpha = g^{-1}\partial_\alpha g$ of a coset element $g(X^\mu) = \exp(iX^\mu P_\mu)$ neither has a Lorentz generator component nor satisfies the M.C. equation for the Poincaré algebra, because the coset algebra is abelian, $[P_\mu, P_\nu] = 0$. So the BIZZ procedure is not applicable naively to a string in a flat space. On the other hand the flat background is obtained by the Inönü-Wigner (IW) contraction of an AdS space represented by the coset of the homogeneous $SO(n,2)$ group over the $SO(n,1)$, then one-form currents obtained by the IW contraction satisfy the Poincaré M.C. equation. For the AdS space before the IW contraction the conserved and "flat" current is nothing but the Noether current of the action. From above two facts we begin with Noether currents as the conserved currents for a flat and pp-wave spaces, and we will examine whether they satisfy the "flatness"

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condition corresponding to the background isometries. Using these conserved and "flat" currents we will construct an infinite set of conserved nonlocal currents and charges in a flat and pp-wave backgrounds.

We also point out how to define the integration path for conserved charges of a closed string with winding modes. For a conserved current $\partial_\alpha J^\alpha = 0$ one utilizes a dual potential χ such that $J_\alpha = \epsilon_{\alpha\beta}\partial^\beta\chi$, but this χ is a single-valued function only on a simply connected region. Therefore we define the potential χ, and hence the nonlocal current, on a semi-infinite strip cut open the cylinderical worldsheet. The conserved charges are given by integration along the boundary path of the semi-infinite strip.

The organization of the paper is as follows. In section 2 the current conservation and the "flatness" condition for a string in a flat background are examined and a general argument of the Noether current for inhomogeneous backgrounds is presented. By the BIZZ procedure we construct nonlocal charges and compare them before and after the T-duality transformation. In section 3 we compute nonlocal charges of a string in the type IIB pp-wave background and write down concrete expressions for zero-th, first and second order. In section 4 T-duality transformation on the Michelson's cycle is performed to obtain the type IIA pp-wave background, and nonlocal charges before and after the T-duality transformation are computed. From the completeness of this correspondence, a conjugate coordinate to the winding mode is introduced by adding the Wess-Zumino term in the type IIB action which corresponds to the Buscher T-duality transformation.

2. Flat background

The action of a string in a flat background has not only the translational symmetry but also the Lorentz symmetry. Although the translational group is abelian, considering the whole Poincaré group gives nontrivial structure of the nonlocal charges especially under T-duality. At first we will clarify the Noether current for inhomogeneous SO(n) cosets and examine the procedure of constructing nonlocal charges. Then we will compute nonlocal charges and compare them under T-duality.

2.1 Noether charges

The action for a string in a flat space is given by

$$S = -\frac{1}{4\pi\alpha'}\int d^2\sigma\ \sqrt{-h}h^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu} \tag{2.1}$$

where $h_{\alpha\beta}$ is the worldsheet metric and the conformal gauge $\sqrt{-h}h^{\alpha\beta} = \eta^{\alpha\beta}$ is chosen from now on. A closed string is quantized as

$$\begin{aligned} X^\mu &= x^\mu + \alpha' p^\mu \tau + \sqrt{\tfrac{\alpha'}{2}}\sum_{n\neq 0}\frac{i}{n}\left(\alpha_n^\mu e^{-in(\tau+\sigma)} + \tilde{\alpha}_n^\mu e^{-in(\tau-\sigma)}\right) \\ [x^\mu, p^\nu] &= i\eta^{\mu\nu}\ \ , \quad [\alpha_m^\mu, \alpha_n^\nu] = m\delta_{m,-n}\eta^{\mu\nu} = [\tilde{\alpha}_m^\mu, \tilde{\alpha}_n^\nu] \qquad . \end{aligned} \tag{2.2}$$

Noether currents under the translation and the Lorentz rotation are

$$P_\alpha^\mu = \tfrac{1}{2\pi\alpha'}\partial_\alpha X^\mu\ \ , \quad M_\alpha^{\mu\nu} = \tfrac{1}{2\pi\alpha'}X^{[\mu}\partial_\alpha X^{\nu]} \tag{2.3}$$

satisfying the conservation law

$$\partial^\alpha P^\mu_\alpha = 0 = \partial^\alpha M^{\mu\nu}_\alpha \,. \tag{2.4}$$

Noether charges are given by

$$\begin{aligned} \hat{P}^\mu &= \int_0^{2\pi} d\sigma \; P^\mu_\tau = p^\mu \\ \hat{M}^{\mu\nu} &= \int_0^{2\pi} d\sigma \; M^{\mu\nu}_\tau = x^{[\mu} p^{\nu]} + \frac{i}{2} \sum_{n\neq 0} \frac{1}{n} \left(\alpha_n^{[\mu} \alpha_{-n}^{\nu]} + \tilde{\alpha}_n^{[\mu} \tilde{\alpha}_{-n}^{\nu]} \right) \end{aligned} \tag{2.5}$$

satisfying the Poincaré algebra

$$\left[\hat{M}^{\mu\nu}, \hat{M}^{\rho\lambda} \right] = i\eta^{[\mu|[\lambda} \hat{M}^{\rho]|\nu]} \;\;, \quad \left[\hat{M}^{\mu\nu}, \hat{P}^\rho \right] = i\eta^{\rho[\mu} \hat{P}^{\nu]} \,. \tag{2.6}$$

The structure constant of the Poincaré algebra in (2.6) is denoted by f^c_{ab} in $[T_a, T_b] = f^c_{ab} T_c$ with $T_a = (\hat{P}^\mu, \; \hat{M}^{\mu\nu})$.

Next let us introduce the one forms by multiplying the one form basis as

$$J^\mu = d\sigma^\alpha P^\mu_\alpha \;\;, \qquad J^{\mu\nu} = d\sigma^\alpha M^{\mu\nu}_\alpha \,, \tag{2.7}$$

then we act the exterior derivative on them

$$dJ^\mu = 0 \;\;, \qquad dJ^{\mu\nu} = 4\pi\alpha' J^\mu \wedge J^\nu \quad . \tag{2.8}$$

In addition to the equations (2.8) the consistency $ddJ^A = 0$ allows to regard the Noether currents (2.3) or (2.7) as currents on a group manifold. If we read off the "structure constant" from (2.8) as

$$dJ^C = \frac{1}{2} F^C_{\rm AB} J^A \wedge J^B \;\;, \quad J^A = (J^\mu, \; J^{\mu\nu}) \quad , \tag{2.9}$$

then this structure constant $F^C_{\rm AB}$ is different from the one of the Poincaré algebra f^c_{ab}. This is common for the inhomogeneous spaces such as a flat space and pp-wave spaces, unlike AdS spaces. At first we explain this feature and then we will use this structure constant $F^C_{\rm AB}$ to obtain the nonlocal charges.

2.2 Noether currents for inhomogeneous SO(n) cosets

Consider a G/H coset sigma model with identification $g(x) \sim g(x)h(x)$, $g(x) \in G$, $h(x) \in H$. Let $\mathcal{G}$ and $\mathcal{H}$ be the Lie algebra of G and H, and let $\mathcal{G} = \mathcal{H} \oplus \mathcal{K}$. We assume that the coset is a symmetric space, that is, there exist a 'parity' transformation θ such that

$$\theta(\mathcal{H}) = +\mathcal{H} \,, \quad \theta(\mathcal{K}) = -\mathcal{K} \,. \tag{2.10}$$

As in [14], define

$$J_\alpha = g^{-1} \partial_\alpha g \,, \quad J_\alpha = H_\alpha + K_\alpha \tag{2.11}$$

with $H_\alpha \in \mathcal{H}$, $K_\alpha \in \mathcal{K}$. The Lagrangian is given by

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4}\mathrm{Tr}\partial_\alpha \mathcal{M}\partial^\alpha \mathcal{M}^{-1}, \qquad \mathcal{M} = g\cdot\theta(g^{-1}) \\ &= \mathrm{Tr}K_\alpha K^\alpha. \end{aligned} \tag{2.12}$$

If the trace is non-degenerate on $\mathcal{K}$, one can derive the equation of motion

$$\partial_\alpha K^\alpha + [H_\alpha, K^\alpha] = 0\,. \tag{2.13}$$

Then it follows that $k_\alpha \equiv gK_\alpha g^{-1}$ is a conserved current:

$$\partial_\alpha k^\alpha = 0\,. \tag{2.14}$$

In general k_α is invariant under the local gauge transformation, $g(x) \to g(x)h(x)$, and is identified as the Noether current of the coset model. The Noether current k_α has both $\mathcal{H}$ and $\mathcal{K}$ components unlike K_α which has only $\mathcal{K}$ components. However, if $\mathcal{K}$ is abelian, k_α belongs to $\mathcal{K}$ and has no $\mathcal{H}$ components. Therefore, in this case k_α constructed above does not exhaust all the Noether currents of the model. In our case the bosonic string action in the flat space is regarded as the coset sigma model of the Poincaré group over the Lorentz group. Since the translation group is abelian, the conserved currents constructed as k_α only contain $\partial_\alpha X^\mu$, the Noether currents for the translation, but not those for the Lorentz rotations.

For a supersymmetric system the coset is not a symmetric space anymore because of the superalgebra $\{Q, Q\} = P$. The 'parity' is modified to four-fold grading [30, 31], and the conserved current is not simply $gK_\alpha g^{-1}$ but with the κ-symmetric modification [16].

Let us consider a flat space as a coset $\mathrm{ISO}(D,1)/\,\mathrm{SO}(D-1,1)$ obtained by the IW contraction of a coset $\mathrm{SO}(D,1)/\,\mathrm{SO}(D-1,1)$. We begin with a coset element $g \in G/H$ with $G = \mathrm{SO}(D,1)$ and $H = \mathrm{SO}(D-1,1)$. The Noether current has both $\mathcal{H}$ and $\mathcal{K}$ components as $k = k_\mathcal{H} + k_\mathcal{K}$ with $k_\mathcal{H} \in \mathcal{H}$ and $k_\mathcal{K} \in \mathcal{K}$. They satisfy the following M.C. equation

$$dk_\mathcal{K} = k_\mathcal{H} \wedge k_\mathcal{K} \ , \quad dk_\mathcal{H} = k_\mathcal{H} \wedge k_\mathcal{H} + k_\mathcal{K} \wedge k_\mathcal{K}\,. \tag{2.15}$$

The IW contraction into the Poincaré algebra reduces to the following M.C. equations

$$k_\mathcal{K} \to \Omega k_\mathcal{K} \ , \quad k_\mathcal{H} \to k_\mathcal{H} \ \text{ and } \ \Omega \to 0 \quad , \ \text{ then } \ dk_\mathcal{K} = k_\mathcal{H} \wedge k_\mathcal{K} \ , \quad dk_\mathcal{H} = k_\mathcal{H} \wedge k_\mathcal{H} \tag{2.16}$$

These are not the equations (2.8). This contraction for the current is realized if a coset element contains auxiliary coordinates $Y^{\mu\nu}$ in addition to the coset parameters X^μ: i.e. a coset element and the one-form currents are functions of X^μ and $Y^{\mu\nu}$. If they are rescaled as $X^\mu \to \Omega X^\mu$ and $Y^{\mu\nu} \to Y^{\mu\nu}$ then the currents can be rescaled as $k_\mathcal{K} \to \Omega k_\mathcal{K}$ and $k_\mathcal{H} \to k_\mathcal{H}$. But if a coset element contains only X^μ then rescaling $X^\mu \to \Omega X^\mu$ forces the currents the following rescaling by the parity requirement

$$k_\mathcal{K} \to \Omega k_\mathcal{K} \ , \quad k_\mathcal{H} \to \Omega^2 k_\mathcal{H} \ \text{ and } \ \Omega \to 0 \ , \ \text{ then } \quad dk_\mathcal{K} = 0 \ , \quad dk_\mathcal{H} = k_\mathcal{K} \wedge k_\mathcal{K} \tag{2.17}$$

These are the equations (2.8), and still represent the Poincaré algebra but in the different gauge. We will use the algebra obtained in (2.17) to construct nonlocal conserved charges. Our criterion is conservation of charges so far. At the end the obtained charges give the correct algebra after the quantization in any case.

2.3 Nonlocal charges

If we introduce the algebra $[T_A, T_B] = F^C_{\rm AB} T_C$ corresponding to the structure coefficient induced by the Noether current $F^C_{\rm AB}$ in (2.8) and (2.9), then the covariant derivative operator can be defined as

$$\mathcal{D}_\alpha = \partial_\alpha - J^A_\alpha T_A \quad \rightarrow \quad [\partial^\alpha, \mathcal{D}_\alpha] = 0 \ \text{ and } \ [\mathcal{D}_\alpha, \mathcal{D}_\beta] = 0\,. \tag{2.18}$$

These equations correspond to the current conservation law (2.4) and the "flatness" condition (2.8) respectively. Then we follow the BIZZ procedure using with this covariant derivative operator in (2.18). An infinite set of conserved nonlocal currents are obtained by the covariant derivative $\mathcal{D}_\alpha$ acting on the dual potential $\chi_{[N]}$'s

$$\mathcal{J}_{[N]\alpha}{}^A = \epsilon_{\alpha\beta}\partial^\beta \chi_{[N]}{}^A \ , \ \ \mathcal{J}_{[N+1]\alpha}{}^A = (\mathcal{D}_\alpha \chi_{[N]})^A \ \ \rightarrow \ \ \partial^\alpha \mathcal{J}_{[N+1]\alpha}{}^A = 0, \ \ {}_{N\geq 0}\,. \tag{2.19}$$

Since $\chi_{[N]}$'s are nonlocal, obtained conserved currents $\mathcal{J}_{[N]}$ are also nonlocal.

Now we will compute the nonlocal charges for a string in the flat background. We list expressions of zero-th, first and second order nonlocal charges here:

$$\mathcal{J}_{[0]\alpha}{}^A = J^A_\alpha = \begin{cases} \frac{1}{2\pi\alpha'}\partial_\alpha X^\mu \\ \frac{1}{2\pi\alpha'} X^{[\mu}\partial_\alpha X^{\nu]} \end{cases} \tag{2.20}$$

$$\mathcal{J}_{[1]\alpha}{}^A = \begin{cases} \frac{1}{2\pi\alpha'}\epsilon_{\alpha\beta}\partial^\beta X^\mu \\ \frac{1}{2\pi\alpha'}\epsilon_{\alpha\beta} X^{[\mu}\partial^\beta X^{\nu]} - \partial_\alpha X^{[\mu}\chi^{\nu]}_{[0]} \end{cases} \tag{2.21}$$

$$\mathcal{J}_{[2]\alpha}{}^A = \begin{cases} \frac{1}{2\pi\alpha'}\partial_\alpha X^\mu \\ \frac{1}{2\pi\alpha'} X^{[\mu}\partial_\alpha X^{\nu]} - (\frac{1}{2\pi\alpha'})^{-1}\left(J^{[\mu}_{[1]\alpha}\chi^{\nu]}_{[0]} + J^{[\mu}_{[0]\alpha}\chi^{\nu]}_{[1]}\right) \end{cases} \tag{2.22}$$

where $\epsilon^{\alpha\beta}\epsilon_{\beta\gamma} = \delta^\alpha_\gamma$ and $\epsilon^{\tau\sigma} = 1 = \epsilon_{\sigma\tau}$.

Let us consider a closed string winding around some compact directions as

$$X^\mu(\sigma + 2\pi) = X^\mu(\sigma) + 2\pi R w^\mu \ , \ \ p^\mu = \frac{n^\mu}{R} \ , \ \ w^\mu, n^\mu \in \mathbf{Z} \ , \tag{2.23}$$

and it is quantized as

$$X^\mu = x^\mu + \alpha' p^\mu \tau + w^\mu R\sigma + \sqrt{\tfrac{\alpha'}{2}}\sum_{n\neq 0}\frac{i}{n}\left(\alpha^\mu_n e^{-in(\tau+\sigma)} + \tilde{\alpha}^\mu_n e^{-in(\tau-\sigma)}\right) \quad .$$

It is required to write $x^\mu = x^\mu_+ + x^\mu_-$ in such a way that they become canonical conjugates of momenta and windings, $\left[x^\mu_+, (p^\nu + w^\nu R/\alpha')/2\right] = i\eta^{\mu\nu} = \left[x^\mu_-, (p^\nu - w^\nu R/\alpha')/2\right]$. It is also noted that there is no restriction on $x^\mu_+ - x^\mu_-$ at this stage.

For the finite range of σ coordinate $0 \leq \sigma \leq 2\pi$ with the nontrivial winding (2.23), a conserved charge is obtained by the integration along the following path (figure 1):

$$Q^A_{[N]} = \int_{-\infty}^{\tau} d\tau' \mathcal{J}_{[N]\sigma}{}^A(\tau', \sigma = 0) + \int_0^{2\pi} d\sigma \mathcal{J}_{[N]\tau}{}^A(\tau, \sigma) + \int_{\tau}^{-\infty} d\tau' \mathcal{J}_{[N]\sigma}{}^A(\tau', \sigma = 2\pi) \tag{2.24}$$

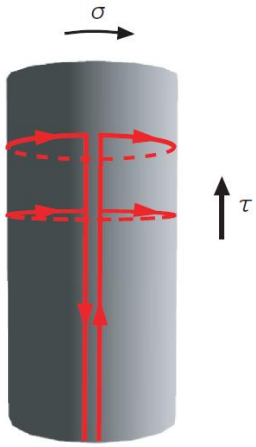

Figure 1: The integration path.

and the dual potential χ defined in (2.19) is computed as

$$\chi^A(\tau,\sigma) = -\int_0^\sigma d\sigma'\ J^A_\tau(\tau,\sigma') - \int_{-\infty}^\tau d\tau'\ J^A_\sigma(\tau',\sigma=0)\,. \tag{2.25}$$

The resultant Noether charges, which are zero-the order charges, (2.5) are again listed here:

$$Q^\mu_{[0]} = \hat{P}^\mu = p^\mu\ ,\quad Q^{\mu\nu}_{[0]} = \hat{M}^{\mu\nu} = x^{[\mu}p^{\nu]} + \frac{i}{2}\sum_{n\neq 0}\frac{1}{n}\left(\alpha_n^{[\mu}\alpha_{-n}^{\nu]} + \tilde{\alpha}_n^{[\mu}\tilde{\alpha}_{-n}^{\nu]}\right) \tag{2.26}$$

The first order nonlocal charges:

$$Q^\mu_{[1]} = -\omega^\mu R\ ,\quad Q^{\mu\nu}_{[1]} = -x^{[\mu}\omega^{\nu]}R - i\sum_{n\neq 0}\frac{1}{n}\left(\alpha_n^{[\mu}\alpha_{-n}^{\nu]} - \tilde{\alpha}_n^{[\mu}\tilde{\alpha}_{-n}^{\nu]}\right) \tag{2.27}$$

The even order, $2N$-th ($N \geq 1$) order nonlocal charges:

$$\begin{aligned} Q^\mu_{[2N]} &= Q^\mu_{[0]} \\ Q^{\mu\nu}_{[2N]} &= Q^{\mu\nu}_{[0]} + N\Delta Q^{\mu\nu}_{[\text{even}]}\ ,\quad \Delta Q^{\mu\nu}_{[\text{even}]} = i\sum_{n\neq 0}\frac{1}{n}\left(\alpha_n^{[\mu}\alpha_{-n}^{\nu]} + \tilde{\alpha}_n^{[\mu}\tilde{\alpha}_{-n}^{\nu]}\right) \end{aligned} \tag{2.28}$$

and for the odd order, $2N+1$-th ($N \geq 1$) order nonlocal charges:

$$\begin{aligned} Q^\mu_{[2N+1]} &= Q^\mu_{[1]} \\ Q^{\mu\nu}_{[2N+1]} &= Q^{\mu\nu}_{[1]} + N\Delta Q^{\mu\nu}_{[\text{odd}]}\ ,\quad \Delta Q^{\mu\nu}_{[\text{odd}]} = -i\sum_{n\neq 0}\frac{1}{n}\left(\alpha_n^{[\mu}\alpha_{-n}^{\nu]} - \tilde{\alpha}_n^{[\mu}\tilde{\alpha}_{-n}^{\nu]}\right) \end{aligned} \tag{2.29}$$

Independent components are zero-th and first order nonlocal charges for zero mode and non-zero mode separately. The momentum, the winding number, the total Lorentz spin and the relative Lorentz spin, which is the difference between the left mover spin and the right mover spin, for zero mode and for non-zero modes are conserved separately.

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2.4 T-duality

T-duality transformation interchanges the momentum and the winding by interchanging $R \leftrightarrow 1/R$. T-duality transformation in $X(\tau, \sigma)$ reduces the interchange $\tau \leftrightarrow \sigma$ which causes

$$n^\mu \leftrightarrow w^\mu \ , \quad R \leftrightarrow 1/R \ , \quad \tilde{\alpha}^\mu_n \leftrightarrow -\tilde{\alpha}^\mu_{-n} \, . \tag{2.30}$$

The Noether charges after the T-duality transformation is given

$$Q^\mu_{[0]} = \tilde{P}^\mu = w^\mu R \ , \quad Q^{\mu\nu}_{[0]} = \tilde{M}^{\mu\nu} = x^{[\mu} w^{\nu]} R + \frac{i}{2} \sum_{n \neq 0} \frac{1}{n} \left(\alpha^{[\mu}_n \alpha^{\nu]}_{-n} - \tilde{\alpha}^{[\mu}_n \tilde{\alpha}^{\nu]}_{-n} \right) . \tag{2.31}$$

These Noether charges correspond to the first order nonlocal charges in the original background (2.27). Further higher order nonlocal charges after T-duality transformation are:

$$\begin{aligned}
&Q^\mu_{[1]} = -p^\mu \ , \quad Q^{\mu\nu}_{[1]} = -x^{[\mu} p^{\nu]} - i \sum_{n \neq 0} \frac{1}{n} \left(\alpha^{[\mu}_n \alpha^{\nu]}_{-n} + \tilde{\alpha}^{[\mu}_n \tilde{\alpha}^{\nu]}_{-n} \right) \\
&Q^\mu_{[2N]} = Q^\mu_{[0]} \ , \quad Q^{\mu\nu}_{[2N]} = Q^{\mu\nu}_{[0]} + N \Delta Q^{\mu\nu}_{[\text{even}]} \ , \quad \Delta Q^{\mu\nu}_{[\text{even}]} = i \sum_{n \neq 0} \frac{1}{n} \left(\alpha^{[\mu}_n \alpha^{\nu]}_{-n} - \tilde{\alpha}^{[\mu}_n \tilde{\alpha}^{\nu]}_{-n} \right) \\
&Q^\mu_{[2N+1]} = Q^\mu_{[1]} \ , \ Q^{\mu\nu}_{[2N+1]} = Q^{\mu\nu}_{[1]} + N \Delta Q^{\mu\nu}_{[\text{odd}]} \ , \ \Delta Q^{\mu\nu}_{[\text{odd}]} = -i \sum_{n \neq 0} \frac{1}{n} \left(\alpha^{[\mu}_n \alpha^{\nu]}_{-n} + \tilde{\alpha}^{[\mu}_n \tilde{\alpha}^{\nu]}_{-n} \right)
\end{aligned} \tag{2.32}$$

The first order nonlocal charges in the T-dualized background correspond to the zero-th order charges of the original background (2.26). The even and the odd order nonlocal charges are interchanged by T-duality. The set of independent conserved charges is equivalent before and after the T-duality transformation in a flat space background.

3. Type IIB pp-wave background

In this section we will compute the nonlocal charges for a closed string in the pp-wave background explicitly. The isometry algebra is inhomogeneous SO(n), so we apply our generalized BIZZ procedure to obtain nonlocal charges. Before introducing nontrivial winding modes for the T-duality transformation we present the nonlocal charges in the type IIB pp-wave background.

3.1 Noether charges

The type IIB pp-wave background is given by

$$\begin{aligned}
ds^2 &= 2dX^+ dX^- + dX^i dX^i - 4\mu^2 X^i X^i (dX^+)^2 \ , \quad {}_{i=1,\ldots,8} \\
{}^{(5)}F &= \mu dX^+ \left(dX^1 dX^2 dX^3 dX^4 + dX^5 dX^6 dX^7 dX^8 \right) .
\end{aligned} \tag{3.1}$$

The action for a string in this background is

$$S = -\frac{1}{4\pi\alpha'} \int d^2\sigma \left[2\partial_\alpha X^+ \partial^\alpha X^- - 4\mu^2 X^i X^i \partial_\alpha X^+ \partial^\alpha X^+ + \partial_\alpha X^i \partial^\alpha X^i \right]_{i=1,\ldots,8} . \tag{3.2}$$

Noether currents of the system are given by:

$$\begin{cases} J^+_\alpha &= \frac{1}{2\pi}\partial_\alpha X^+ \\ J^-_\alpha &= \frac{1}{2\pi\alpha'}\left(\partial_\alpha X^- - 4\mu^2 X^i X^i \partial_\alpha X^+\right) \\ J^i_\alpha &= \frac{1}{2\pi\alpha'}\left(\partial_\alpha X^i \cos 2\mu X^+ - X^i \partial_\alpha \cos 2\mu X^+\right) \\ J^{i*}_\alpha &= \frac{1}{4\pi\mu}\left(\partial_\alpha X^i \sin 2\mu X^+ - X^i \partial_\alpha \sin 2\mu X^+\right) \\ J^{ij}_\alpha &= \frac{1}{2\pi\alpha'} X^{[i}\partial_\alpha X^{J]} \end{cases} \tag{3.3}$$

In the lightcone gauge $X^+ = p^+\tau$ the action becomes

$$S = -\frac{1}{4\pi\alpha'}\int d^2\sigma \left[\partial_\alpha X^i \partial^\alpha X^i + \hat{\mu}^2 X^i X^i\right] \quad , \quad \hat{\mu} = 2\mu p^+ \quad , \quad {}_{i=1,\ldots,8} \tag{3.4}$$

and Virasoro constraints are

$$\begin{cases} h = 2\mathcal{P}^- p^+ + \dot{X}^i \dot{X}^i + X^{i'} X^{i'} + \hat{\mu}^2 X^i X^i = 0 \\ t = p^+ X^{-'} + \dot{X}^i X^{i'} = 0 \end{cases} \tag{3.5}$$
$$\mathcal{P}^- = \dot{X}^- - 4\mu^2 p^+ X^i X^i$$

with $\dot{X} = \partial_\tau X$ and $X' = \partial_\sigma X$. The quantization in the lightcone gauge is performed as

$$X^i = x^i \cos\hat{\mu}\tau + p^i \frac{\alpha'}{\hat{\mu}} \sin\hat{\mu}\tau + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{1}{n\rho_n}\left(\alpha^i_n e^{-in\sigma} + \tilde{\alpha}^i_n e^{in\sigma}\right) e^{-in\rho_n\tau}$$
$$\rho_n = \sqrt{1 + \left(\frac{\hat{\mu}}{n}\right)^2} \tag{3.6}$$

with

$$\left[x^i, p^j\right] = i\delta^{ij} \quad , \quad \left[\alpha^i_n, \alpha^j_m\right] = n\rho_n \delta^{ij}\delta_{n,-m} = \left[\tilde{\alpha}^i_n, \tilde{\alpha}^j_m\right] . \tag{3.7}$$

Noether charges are given as

$$\begin{cases} Q^+_{[0]} = Q^+ = p^+ \\ Q^-_{[0]} = Q^- = -\frac{1}{2p^+}\left[\alpha' p^i p^i + \frac{\hat{\mu}^2}{\alpha'} x^i x^i + \sum_{n\neq 0}(\alpha^i_n \alpha^i_{-n} + \tilde{\alpha}^i_n \tilde{\alpha}^i_{-n})\right] \\ Q^i_{[0]} = Q^i = p^i \\ Q^{i*}_{[0]} = Q^{i*} = -p^+ x^i \\ Q^{ij}_{[0]} = Q^{ij} = x^{[i} p^{j]} + \frac{i}{2}\sum_{n\neq 0} \frac{1}{n\rho_n}(\alpha^{[i}_n \alpha^{j]}_{-n} + \tilde{\alpha}^{[i}_n \tilde{\alpha}^{j]}_{-n}) \end{cases} \tag{3.8}$$

which satisfy the following pp-wave algebra:

$$\begin{array}{lll} \left[Q^-, Q^i\right] = i\frac{4\mu^2}{\alpha'} Q^{i*} & , \left[Q^-, Q^{i*}\right] = -i\alpha' Q^i & , \left[Q^i, Q^{j*}\right] = i\delta^{ij} Q^+ \\ \left[Q^{ij}, Q^k\right] = -i\delta^{k[i} Q^{j]} & , \left[Q^{ij}, Q^{k*}\right] = -i\delta^{k[i} Q^{j]*} & , \left[Q^{ij}, Q^{kl}\right] = -i\delta^{[i|[l} Q^{k]|j]} \end{array} \tag{3.9}$$
$$\text{others} = 0$$

3.2 Nonlocal charges

One form currents constructed from the Noether currents (3.3) as $J = d\sigma^\alpha J_\alpha$ satisfy the following equations:

$$\begin{cases} dJ^+ = 0 & , \ dJ^- = 16\pi\mu^2 J^i \wedge J^{i*} \\ dJ^i = -\dfrac{16\pi\mu^2}{\alpha'} J^+ \wedge J^{i*} & , \ dJ^{i*} = 4\pi\alpha' J^+ \wedge J^i \\ dJ^{ij} = 4\pi\alpha' \left(J^i \wedge J^j + \dfrac{4\mu^2}{\alpha'^2} J^{i*} \wedge J^{j*} \right) & \end{cases} \tag{3.10}$$

They are "flatness" condition and the consistency condition $ddJ = 0$ is also satisfied, so the covariant derivative operator (2.18) can be constructed. The structure constant read off from the above equations (3.10) is different from the one of the pp-wave algebra (3.9) as expected in the subsection 2.1.

According to the procedure (2.19) the first order nonlocal currents are given by

$$\begin{cases} \mathcal{J}_{[1]}{}^+_\alpha &= \epsilon_{\alpha\beta} J^{+\beta} \\ \mathcal{J}_{[1]}{}^-_\alpha &= \epsilon_{\alpha\beta} J^{-\beta} - 8\pi\mu^2 \left(J^i_\alpha \chi^{i*}_{[0]} - J^{i*}_\alpha \chi^i_{[0]} \right) \\ \mathcal{J}_{[1]}{}^i_\alpha &= \epsilon_{\alpha\beta} J^{i\beta} + \frac{8\pi\mu^2}{\alpha'} \left(J^+_\alpha \chi^{i*}_{[0]} - J^{i*}_\alpha \chi^+_{[0]} \right) \\ \mathcal{J}_{[1]}{}^{i*}_\alpha &= \epsilon_{\alpha\beta} J^{i*\beta} - 2\pi\alpha' \left(J^+_\alpha \chi^i_{[0]} - J^i_\alpha \chi^+_{[0]} \right) \\ \mathcal{J}_{[1]}{}^{ij}_\alpha &= \epsilon_{\alpha\beta} J^{ij\beta} - 2\pi\alpha' \left(J^{[i}_\alpha \chi^{j]}_{[0]} + \dfrac{4\mu^2}{\alpha'^2} J^{[i*}_\alpha \chi^{j]*}_{[0]} \right) \end{cases} \tag{3.11}$$

where the dual potential $\chi_{[0]}$'s are given as:

$$\chi^+_{[0]} = -\frac{1}{2\pi} p^+ \sigma \tag{3.12}$$

$$\chi^i_{[0]} = -\frac{1}{2\pi} p^i \sigma - \frac{i}{4\pi\sqrt{2\alpha'}} \sum_{n\neq 0} \frac{1}{n} \left(\left(1 + \frac{\hat{\mu}}{n\rho_n} \right) e^{i\hat{\mu}\tau} + \left(1 - \frac{\hat{\mu}}{n\rho_n} \right) e^{-i\hat{\mu}\tau} \right) \left(\alpha^i_n e^{-in\sigma} - \tilde{\alpha}^i_n e^{in\sigma} \right) e^{-in\rho_n \tau}$$

$$\chi^{i*}_{[0]} = \frac{1}{2\pi} p^+ x^i \sigma - \frac{\sqrt{\alpha'}}{8\pi\mu\sqrt{2}} \sum_{n\neq 0} \frac{1}{n} \left(\left(1 + \frac{\hat{\mu}}{n\rho_n} \right) e^{i\hat{\mu}\tau} - \left(1 - \frac{\hat{\mu}}{n\rho_n} \right) e^{-i\hat{\mu}\tau} \right) \left(\alpha^i_n e^{-in\sigma} - \tilde{\alpha}^i_n e^{in\sigma} \right) e^{-in\rho_n \tau}$$

The first order nonlocal charges are obtained as:

$$\begin{cases} Q^-_{[1]} &= \dfrac{1}{2p^+} \displaystyle\sum_{n\neq 0} \frac{1}{\rho_n} \left(1 + 2 \left(\frac{\hat{\mu}}{n} \right)^2 \right) \left(\alpha^i_n \alpha^i_{-n} - \tilde{\alpha}^i_n \tilde{\alpha}^i_{-n} \right) \\ Q^{ij}_{[1]} &= i \displaystyle\sum_{n\neq 0} \frac{1}{n} \left(\alpha^{[i}_n \alpha^{j]}_{-n} - \tilde{\alpha}^{[i}_n \tilde{\alpha}^{j]}_{-n} \right) \\ \text{others} &= 0 \end{cases} \tag{3.13}$$

where $X^{-'}$ in $\mathcal{J}^-_{[1]\tau}$ is determined from the Virasoro condition $t = 0$ in (3.5). In the ref. [26] this term was set to be zero because of the total derivative of σ, but there is a non-zero contribution determined by the Virasoro condition.

The second order nonlocal current is given by

$$\mathcal{J}_{[2]}{}^A_\alpha = J^A_\alpha - \frac{1}{2} F^A_{BC} \left(\epsilon_{\alpha\beta} J^{B\beta} \chi^C_{[0]} + J^B_\alpha \chi^C_{[1]} \right) \tag{3.14}$$

and the second order nonlocal charges are obtained as

$$\begin{cases} Q^{-}_{[2]} - Q^{-}_{[0]} = -\dfrac{1}{p^+}\displaystyle\sum_{n\neq 0}\left(\frac{\hat{\mu}}{n\rho_n}\right)^2\left(1+2\left(\frac{\hat{\mu}}{n}\right)^2\right)\left(\alpha^i_n\alpha^i_{-n}+\tilde{\alpha}^i_n\tilde{\alpha}^i_{-n}\right) \\ Q^{ij}_{[2]} - Q^{ij}_{[0]} = i\displaystyle\sum_{n\neq 0}\frac{1}{n\rho_n}\left(1+2\left(\frac{\hat{\mu}}{n}\right)^2\right)\left(\alpha^{[i}_n\alpha^{j]}_{-n}+\tilde{\alpha}^{[i}_n\tilde{\alpha}^{j]}_{-n}\right) \\ \quad \text{others} \quad = 0 \end{cases} \tag{3.15}$$

These charges (3.15) are different from $Q_{[0]}$ in (3.8). Higher order nonlocal charges are obtained by this computation by the procedure (2.19), and it is expected that coefficients on the nonlocal charges are different in each order unlike the flat case (2.28) and (2.29). There exist an infinite number of independent conserved charges exist in the pp-wave background contrast to the flat case.

4. T-dual pp-wave backgrounds

Now we will examine the nonlocal charges for closed strings with a winding mode in the pp-wave backgrounds. We will compare the nonlocal charges before and after the T-duality transformation.

4.1 Michelson's cycle

In order to examine T-duality a spacelike circle is needed to compactify it. First we rewrite IIB pp-wave coordinate in (3.1) in terms of x^μ, then change variables as

$$\begin{cases} x^+ &= X^+ \\ x^- &= X^- - 2\mu X^1 X^2 \\ x^I &= X^I \quad , \quad I{=}3,\ldots,8 \\ \begin{pmatrix} x^1 \\ x^2 \end{pmatrix} &= \begin{pmatrix} \cos 2\mu X^+ & -\sin 2\mu X^+ \\ \sin 2\mu X^+ & \cos 2\mu X^+ \end{pmatrix}\begin{pmatrix} X^1 \\ X^2 \end{pmatrix} \end{cases} \tag{4.1}$$

to get

$$\begin{aligned} ds^2 &= 2dX^+dX^- - 4\mu^2 X^I X^I (dX^+)^2 + dX^i dX^i - 8\mu dX^+ X^2 dX^1 \quad , \quad i{=}1,\ldots,8 \quad I{=}3,\ldots,8 \\ {}^{(5)}F &= \mu dX^+\left(dX^1dX^2dX^3dX^4 + dX^5dX^6dX^7dX^8\right) . \end{aligned} \tag{4.2}$$

The action of a string in the Michelson's pp-wave geometry is given by

$$\begin{aligned} &S_{\rm IIB} = \\ &= -\frac{1}{4\pi\alpha'}\int d^2\sigma\left[2\partial_\alpha X^+\partial^\alpha X^- - 4\mu^2 X^I X^I \partial_\alpha X^+\partial^\alpha X^+ + \partial_\alpha X^i\partial^\alpha X^i - 8\mu X^2\partial_\alpha X^1\partial^\alpha X^+\right] \\ &\quad i{=}1,2,3,\ldots,8 \quad , \quad I{=}3,\ldots,8\,. \end{aligned} \tag{4.3}$$

Target space indices $-, 1, 2$ are renamed in such a way that they coincide translations and rotations of the new coordinate basis (4.2). We compactify a spacelike S^1 along the cycle S^+_{12} whose isometry $k_{S^+_{12}} = k_{e_1} + \frac{1}{2\mu}k_{e^*_2}$. It breaks symmetry from SO(4) × SO(4) into

$\mathrm{SO}(2) \times \mathrm{SO}(4)$, so $1+6$ isometries survive. Some translations and boosts are also broken, and we list survived isometries in Michelson's notation corresponding to our notation up to normalization:

$$\begin{aligned}
k_{e_-} &\Leftrightarrow \mathfrak{k}^+ = \tfrac{\partial}{\partial X^-} \\
k_{e_+} - 2\mu k_{M_{12}} &\Leftrightarrow \mathfrak{k}^- = \tfrac{\partial}{\partial X^+} \\
k_{e_1} + \tfrac{1}{2\mu} k_{e_2^*} = k_{S_{12}^+} &\Leftrightarrow \mathfrak{k}^1 = \tfrac{\partial}{\partial X^1} \\
k_{e_2} + \tfrac{1}{2\mu} k_{e_1^*} = k_{S_{21}^+} &\Leftrightarrow \mathfrak{k}^2 = (\cos 4\mu X^+)\tfrac{\partial}{\partial X^2} + (\sin 4\mu X^+)\tfrac{\partial}{\partial X^1} + 4\mu(\sin 4\mu X^+)X^2\tfrac{\partial}{\partial X^-} \\
k_{e_1} - \tfrac{1}{2\mu} k_{e_2^*} = k_{S_{12}^-} &\Leftrightarrow \mathfrak{k}^{2*} = (\cos 4\mu X^+)\tfrac{\partial}{\partial X^1} - (\sin 4\mu X^+)\tfrac{\partial}{\partial X^2} + 4\mu(\cos 4\mu X^+)X^2\tfrac{\partial}{\partial X^-} \\
k_{e_I} &\Leftrightarrow \mathfrak{k}^I = (\cos 2\mu X^+)\tfrac{\partial}{\partial X^I} + 2\mu(\sin 4\mu X^+)X^I\tfrac{\partial}{\partial X^-} \\
k_{e_I^*} &\Leftrightarrow \mathfrak{k}^{I*} = (\sin 2\mu X^+)\tfrac{\partial}{\partial X^I} - 2\mu(\cos 4\mu X^+)X^I\tfrac{\partial}{\partial X^-}
\end{aligned} \tag{4.4}$$

A Killing vector,

$$k_{S_{21}^-} = k_{e_2} - \tfrac{1}{2\mu} k_{e_1^*} \Leftrightarrow \mathfrak{k}^{1*} = \tfrac{\partial}{\partial X^2} + 4\mu X^1 \tfrac{\partial}{\partial X^-} \ , \tag{4.5}$$

becomes multivalued after compactification of X^1 direction so ill-defined. There are 17 isometries survived and they are noncompact Killing vectors except $k_{S_{12}^+}$. This geometry is totally $1+6+17=24$ dimensional group. The Noether currents, J_α^A, corresponding to the Killing vectors $\mathfrak{k}^A$ are given as:

$$\left\{\begin{array}{ll}
J_\alpha^+ &= \frac{1}{2\pi}\partial_\alpha X^+ \\
J_\alpha^- &= \frac{1}{2\pi\alpha'}\left(\partial_\alpha X^- - 4\mu^2 X^I X^I \partial_\alpha X^+ - 4\mu X^2 \partial_\alpha X^1\right) \\
J_\alpha^1 &= \frac{1}{2\pi\alpha'}\left(\partial_\alpha X^1 - 4\mu X^2 \partial_\alpha X^+\right) \\
J_\alpha^{1*} &= \frac{1}{4\pi\mu}\left(\partial_\alpha X^2 + 4\mu X^1 \partial_\alpha X^+\right) \\
J_\alpha^2 &= \frac{1}{2\pi\alpha'}\left(\partial_\alpha X^2 \ \cos 4\mu X^+ + \partial_\alpha X^1 \ \sin 4\mu X^+\right) \\
J_\alpha^{2*} &= \frac{1}{4\pi\mu}\left(\partial_\alpha X^2 \ \sin 4\mu X^+ - \partial_\alpha X^1 \ \cos 4\mu X^+\right) \\
J_\alpha^I &= \frac{1}{2\pi\alpha'}\left(\partial_\alpha X^I \ \cos 2\mu X^+ - X^I \partial_\alpha \cos 2\mu X^+\right) \\
J_\alpha^{I*} &= \frac{1}{4\pi\mu}\left(\partial_\alpha X^I \ \sin 2\mu X^+ - X^I \partial_\alpha \sin 2\mu X^+\right) \\
J_\alpha^{\mathrm{IJ}} &= \frac{1}{2\pi\alpha'} X_{[I}\partial_\alpha X_{J]}
\end{array}\right. \tag{4.6}$$

where J_α^{1*} is the one for the non-compact case although it will be compactified soon later.

In the lightcone gauge the action for a string in the Michelson's pp-wave geometry becomes

$$\begin{aligned}
S_{\mathrm{IIB}} = -\frac{1}{4\pi\alpha'}\int d^2\sigma \ \left[\partial_\alpha X^i \partial^\alpha X^i + \hat{\mu}^2 X^I X^I + 4\hat{\mu} X^2 \dot{X}^1\right] \\
\hat{\mu} = 2p^+\mu \ , \quad {}_{i=1,2,3,\ldots,8} \ , \quad {}_{I=3,\ldots,8} \ ,
\end{aligned} \tag{4.7}$$

and the Virasoro constraints are

$$\left\{\begin{array}{l}
h_{\mathrm{IIB}} = 2\mathcal{P}^- p^+ + \dot{X}^i \dot{X}^i + X^{i'} X^{i'} + \hat{\mu}^2 X^I X^I = 0 \\
t_{\mathrm{IIB}} = p^+ X^{-'} + \left(\dot{X}^1 - 2\hat{\mu} X^2\right) X^{1'} + \dot{X}^2 X^{2'} + \dot{X}^I X^{I'} = 0
\end{array}\right. \tag{4.8}$$

$$\mathcal{P}^- = \dot{X}^- - 4\mu^2 p^+ X^I X^I - 4\mu X^2 \dot{X}^1 \, .$$

A closed string with a winding mode $X^1(\sigma+2\pi) = X^1(\sigma)+2\pi Rw$ can be quantized as

$$
\begin{aligned}
X^I &= x^I \cos\hat{\mu}\tau + \frac{\alpha'}{\hat{\mu}} p^I \sin\hat{\mu}\tau + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{1}{n\rho_n} \left(\alpha_n^I e^{-in\sigma} + \tilde{\alpha}_n^I e^{+in\sigma}\right) e^{-in\rho_n\tau} \\
X^1 + iX^2 &= wR\sigma + e^{-i\hat{\mu}\tau} \left\{ x\cos\hat{\mu}\tau + \frac{\alpha'}{\hat{\mu}} p \sin\hat{\mu}\tau + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{1}{n\rho_n} \left(\alpha_n e^{-in\sigma} + \tilde{\alpha}_n e^{in\sigma}\right) e^{-in\rho_n\tau} \right\} \\
X^1 - iX^2 &= wR\sigma + e^{i\hat{\mu}\tau} \left\{ \bar{x}\cos\hat{\mu}\tau + \frac{\alpha'}{\hat{\mu}} \bar{p} \sin\hat{\mu}\tau + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{1}{n\rho_n} \left(\bar{\alpha}_n e^{-in\sigma} + \tilde{\bar{\alpha}}_n e^{in\sigma}\right) e^{-in\rho_n\tau} \right\} \\
\rho_n &= \sqrt{1+\frac{\hat{\mu}^2}{n^2}}
\end{aligned}
\tag{4.9}
$$

with

$$
\begin{aligned}
\left[x^I, p^J\right] &= i\delta^{\mathrm{IJ}} \quad , \quad [x, \bar{p}] = i = [\bar{x}, p] \\
\left[\alpha_n^I, \alpha_m^J\right] &= n\rho_n \delta^{\mathrm{IJ}} \delta_{n,-m} = \left[\tilde{\alpha}_n^I, \tilde{\alpha}_m^J\right] \\
[\alpha_n, \bar{\alpha}_m] &= 2n\rho_n \delta_{n,-m} = [\tilde{\alpha}_n, \tilde{\bar{\alpha}}_m] \ .
\end{aligned}
\tag{4.10}
$$

The Noether charges, which are the zero-th order charges, are followings:

$$
\left\{
\begin{aligned}
Q_{[0]}^+ &= p^+ \\
Q_{[0]}^- &= -\tfrac{1}{2p^+} \left[\sum_{i=2,3,\ldots,8} \left(\alpha' p^i p^i + \frac{\hat{\mu}^2}{\alpha'} x^i x^i \right) + \frac{1}{\alpha'} (wR)^2 \right. \\
&\qquad \left. + \sum_{n\neq 0} \left\{ (\alpha_n^I \alpha_{-n}^I + \tilde{\alpha}_n^I \tilde{\alpha}_{-n}^I) + \left(1 + \frac{\hat{\mu}}{n\rho_n}\right) (\alpha_n \bar{\alpha}_{-n} + \tilde{\alpha}_n \tilde{\bar{\alpha}}_{-n}) \right\} \right] \\
Q_{[0]}^1 &= p^1 \\
Q_{[0]}^{1*} &= p^+ \left(x^1 + 2\pi wR\right) \\
Q_{[0]}^2 &= p^2 \\
Q_{[0]}^{2*} &= p^+ x^2 \\
Q_{[0]}^I &= p^I \\
{Q_{[0]}^I}^* &= -p^+ x^I \\
Q_{[0]}^{\mathrm{IJ}} &= -x^{[I} p^{J]} + \tfrac{i}{2} \sum_{n\neq 0} \frac{1}{n\rho_n} \left(\alpha_n^{[I} \alpha_{-n}^{J]} + \tilde{\alpha}_n^{[I} \tilde{\alpha}_{-n}^{J]} \right)
\end{aligned}
\right.
\tag{4.11}
$$

where zero mode variables are rewritten as

$$
\begin{cases}
p^1 = \frac{1}{2}(p+\bar{p}) + i\frac{\hat{\mu}}{2\alpha'}(x-\bar{x}) \ , \ x^1 = \frac{1}{2}(x+\bar{x}) - i\frac{\alpha'}{2\hat{\mu}}(p-\bar{p}) \\
p^2 = \frac{1}{2i}(p-\bar{p}) - \frac{\hat{\mu}}{2\alpha'}(x+\bar{x}) \ , \ x^2 = \frac{i}{2}(x-\bar{x}) - \frac{\alpha'}{2\hat{\mu}}(p+\bar{p})
\end{cases}
\tag{4.12}
$$

$$
\left[x^1, p^1\right] = i = \left[x^2, p^2\right] \ . \tag{4.13}
$$

The above charges excluding Q^1 and Q^{1*} make a closed algebra. After compactify X^1 direction as $X^1 \sim X^1 + 2\pi wR$ the winding mode breaks $Q_{[0]}^{1*}$ symmetry of the vacuum.

The one form currents made from the Noether currents in (4.11) as $J^A = d\sigma^\alpha J^A_\alpha$ satisfy the following M.C. equations:

$$\begin{cases} dJ^+ = 0, & dJ^- = 16\pi\mu^2 \left(J^2 \wedge J^{2*} + J^I \wedge J^{I*}\right) \\ dJ^1 = \frac{16\pi\mu^2}{\alpha'} J^+ \wedge J^{1*}, & dJ^{1*} = -4\pi\alpha' J^+ \wedge J^1 \\ dJ^2 = -\frac{16\pi\mu^2}{\alpha'} J^+ \wedge J^{2*}, & dJ^{2*} = 4\pi\alpha' J^+ \wedge J^2 \\ dJ^I = -\frac{16\pi\mu^2}{\alpha'} J^+ \wedge J^{I*}, & dJ^{I*} = 4\pi\alpha' J^+ \wedge J^I \\ dJ^{\mathrm{IJ}} = 4\pi\alpha' \left(J^I \wedge J^J + \frac{4\mu^2}{\alpha'^2} J^{I*} \wedge J^{J*}\right) & \end{cases} \tag{4.14}$$

The structure constant read off from these M.C. equations is different from the algebra generated by charges (4.11) again. The consistency $ddJ = 0$ corresponding to the Jacobi identity is satisfied without J^1 and J^{1*}. Then compactification of X^1-direction is possible consistently. The first order conserved nonlocal charges are obtained as follows:

$$\begin{cases} Q^1_{[1]} &= -\frac{wR}{\alpha'} \\ Q^-_{[1]} &= \frac{1}{2p^+} \sum_{n\neq 0} \frac{1}{\rho_n} \left\{ \left(1 + 2\left(\frac{\hat{\mu}}{n}\right)^2\right) (\alpha^I_n \alpha^I_{-n} - \tilde{\alpha}^I_n \tilde{\alpha}^I_{-n}) \right. \\ & \qquad \left. + \left(\rho_n + \frac{\hat{\mu}}{n}\right)^2 (\alpha_n \bar{\alpha}_{-n} - \tilde{\alpha}_n \tilde{\bar{\alpha}}_{-n}) \right\} \\ Q^{\mathrm{ij}}_{[1]} &= i \sum_{n\neq 0} \frac{1}{n} \left(\alpha^{[i}_n \alpha^{j]}_{-n} - \tilde{\alpha}^{[i}_n \tilde{\alpha}^{j]}_{-n}\right) \\ \text{others} &= 0 \end{cases} \tag{4.15}$$

It is essential to use the integration path in (2.24) to obtain consistent charges especially $Q^-_{[1]}, Q^2_{[1]}, Q^{2*}_{[1]}$.

4.2 T-duality

T-duality of the type IIB nine dimensional geometry (4.2) along X^1-direction transforms into the type IIA background . For the pp-wave background the NS/NS two-form potential B and the R/R three form potential$^{(3)}C$ of the type IIA theory are responsible for the nontrivial geometry the type IIB theory,

$$\begin{aligned} ds^2 &= 2dX^+dX^- - 4\mu^2 \left(X^I X^I + 4(X^2)^2\right)(dX^+)^2 + dX^i dX^i \ , \quad {}_{i=1,2,\ldots,8, \quad I=3,\ldots,8} \\ {}^{(3)}C &= 8\mu X^+ dX^2 dX^3 dX^4 \ , \quad B = -4\mu X^2 dX^1 dX^+ \, . \end{aligned} \tag{4.16}$$

The action for a string in this IIA background is

$$\begin{aligned} S_{\mathrm{IIA}} = -\frac{1}{4\pi\alpha'} \int d^2\sigma \ \big[& 2\partial_\alpha X^+ \partial^\alpha X^- - 4\mu^2 \left(X^I X^I + 4(X^2)^2\right) \partial_\alpha X^+ \partial^\alpha X^+ \\ & + \partial_\alpha X^i \partial^\alpha X^i + 4\mu X^2 \epsilon^{\alpha\beta} \partial_\alpha X^1 \partial_\beta X^+ \big] \, . \end{aligned} \tag{4.17}$$

The Noether currents in the IIA pp-wave background are given as:

$$\begin{cases} J^+_\alpha &= \frac{1}{2\pi}\partial_\alpha X^+ \\ J^-_\alpha &= \frac{1}{2\pi\alpha'}\left\{\partial_\alpha X^- - 4\mu^2\left(X^I X^I + 4(X^2)^2\right)\partial_\alpha X^+ - 2\mu X^2\epsilon_{\alpha\beta}\partial^\beta X^1\right\} \\ J^1_\alpha &= \frac{1}{2\pi\alpha'}\left(\partial_\alpha X^1 + 4\mu X^2\epsilon_{\alpha\beta}\partial^\beta X^+\right) \\ J^{1*}_\alpha &= \frac{1}{2\pi}\left(X^+\partial_\alpha X^1 - X^1\partial_\alpha X^+ + 2\mu X^2\epsilon_{\alpha\beta}\partial^\beta (X^+)^2\right) \\ J^2_\alpha &= \frac{1}{4\pi\alpha'}\left(\partial_\alpha X^2\ \cos 4\mu X^+ - X^2\partial_\alpha \cos 4\mu X^+\right) \\ J^{2*}_\alpha &= -\frac{1}{8\pi\mu}\left(\partial_\alpha X^2\ \sin 4\mu X^+ - X^2\partial_\alpha \sin 4\mu X^+\right) \\ J^I_\alpha &= \frac{1}{2\pi\alpha'}\left(\partial_\alpha X^I\ \cos 2\mu X^+ - X^I\partial_\alpha \cos 2\mu X^+\right) \\ J^{I*}_\alpha &= \frac{1}{4\pi\mu}\left(\partial_\alpha X^I\ \sin 2\mu X^+ - X^I\partial_\alpha \sin 2\mu X^+\right) \\ J^{\mathrm{IJ}}_\alpha &= \frac{1}{2\pi\alpha'}X^{[I}\partial_\alpha X^{J]} \end{cases} \tag{4.18}$$

In the lightcone gauge the action becomes

$$S_{\mathrm{IIA}} = -\frac{1}{4\pi\alpha'}\int d^2\sigma\ \left[\partial_\alpha X^i\partial^\alpha X^i - \hat{\mu}^2\left(X^I X^I + 4(X^2)^2\right) - 4\hat{\mu}X^2X^{1\prime}\right] \quad , \tag{4.19}$$

and the Virasoro constraints are

$$\begin{cases} h_{\mathrm{IIA}} &= 2\mathcal{P}^- p^+ + \dot{X}^i\dot{X}^i + X^{i\prime}X^{i\prime} + \hat{\mu}^2 X^I X^I + 4\hat{\mu}^2(X^2)^2 + 4\hat{\mu}X^2X^{1\prime} = 0 \\ t_{\mathrm{IIA}} &= p^+X^{-\prime} + \dot{X}^i X^{i\prime} = 0 \end{cases} \tag{4.20}$$
$$\mathcal{P}^- = \dot{X}^- - 4\mu^2 p^+\left(X^I X^I + 4(X^2)^2\right) - 4\mu X^2 X^{1\prime}\,.$$

The quantization is given by

$$\begin{aligned} X^1 &= w^\natural R\sigma + x^\natural + \alpha' p^\natural\tau \\ &\quad + \frac{\sqrt{\alpha'}}{2\sqrt{2}}\sum_{n\neq 0}\frac{1}{n}\left\{(-1+\frac{\hat{\mu}}{n\rho_n})(\alpha_n e^{-in\sigma} + \tilde{\alpha}_n e^{in\sigma})e^{-i\hat{\mu}\tau}\right. \\ &\qquad \left. + (1+\frac{\hat{\mu}}{n\rho_n})(\bar{\alpha}_n e^{-in\sigma} + \tilde{\bar{\alpha}}_n e^{in\sigma})e^{i\hat{\mu}\tau}\right\} e^{-in\rho_n\tau} \\ X^2 &= -\frac{w^\natural R}{2\hat{\mu}} + x^2\cos 2\hat{\mu}\tau + \frac{\alpha'}{\hat{\mu}}p^2\sin 2\hat{\mu}\tau \\ &\quad + \frac{\sqrt{\alpha'}}{2\sqrt{2}}\sum_{n\neq 0}\frac{i}{n\rho_n}\left\{(\alpha_n e^{-in\sigma} - \tilde{\alpha}_n e^{in\sigma})e^{-i\hat{\mu}\tau} + (\bar{\alpha}_n e^{-in\sigma} - \tilde{\bar{\alpha}}_n e^{in\sigma})e^{i\hat{\mu}\tau}\right\} e^{-in\rho_n\tau} \end{aligned} \tag{4.21}$$

with $\left[x^\natural, p^\natural\right] = i$ in addition to X^I in the (4.9).

The Noether charges in the IIA background are given as

$$\begin{cases} Q^1_{[0]} &= p^\natural \\ Q^{1*}_{[0]} &= -p^+(x^\natural + Rw^\natural\pi) \\ Q^-_{[0]} &= -\frac{1}{2p^+}\left[\sum_{i=2,3,\ldots,8}\left(\alpha' p^i p^i + \frac{\hat{\mu}^2}{\alpha'}x^i x^i\right) + \alpha'(p^\natural)^2\right. \\ &\qquad \left. + \sum_{n\neq 0}\left\{(\alpha^I_n\alpha^I_{-n} + \tilde{\alpha}^I_n\tilde{\alpha}^I_{-n}) + \left(1+\frac{\hat{\mu}}{n\rho_n}\right)(\alpha_n\bar{\alpha}_{-n} + \tilde{\alpha}_n\tilde{\bar{\alpha}}_{-n})\right\}\right] \end{cases} \tag{4.22}$$

and $Q^2_{[0]}, Q^{2*}_{[0]}, Q^I_{[0]}, Q^{I*}_{[0]}, Q^{\rm IJ}_{[0]}$ are the same with the IIB case in (4.11). Comparing the lightcone Hamiltonian in the type IIA background, $Q^-_{[0]}$, in (4.22) with the one in the type IIB background in (4.11) leads to

$$Q^-_{[0]\rm IIA} = Q^-_{[0]\rm IIB} \quad \Leftrightarrow \quad \frac{1}{\alpha'}(wR)^2 = \alpha'(p^\natural)^2 \,. \tag{4.23}$$

Then the zero-th order momentum charge $Q^1_{[0]}$ in the IIA background corresponds to the first order nonlocal momentum charge $Q^1_{[1]}$ in the IIB background in (4.15)

$$Q^1_{[0]\rm IIA} = -Q^1_{[1]\rm IIB} \,. \tag{4.24}$$

In order to make this correspondence complete as

$$Q^{1*}_{[0]\rm IIA} = -Q^{1*}_{[1]\rm IIB} \quad , \tag{4.25}$$

let us add a Wess-Zumino term in the "IIB" action

$$\mathcal{L}_{IIB;WZ} = \frac{1}{2} X^\natural \epsilon^{\alpha\beta}(\partial_\alpha J^1_\beta + \frac{4\mu}{2\pi\alpha'}\partial_\alpha X^2 \partial_\beta X^+) \tag{4.26}$$

where $X^\natural$ is a Lagrange multiplier to ensure the M.C. equation of J^1 in (4.14). Under the $\mathfrak{k}^{1*} = \frac{\partial}{\partial X^2} + 4\mu X^1 \frac{\partial}{\partial X^-}$ transformation the variation of $\mathcal{L}_{IIB;WZ}$ gives a new contribution to the current $J^{1*}_{\rm IIB}$ as

$$\delta_{\lambda^{1*}}\mathcal{L}_{IIB;WZ} = \partial^\alpha \lambda^{1*} \Delta J^{1*}_\alpha \quad , \quad \Delta J^{1*}_\alpha = -\frac{1}{2\pi} X^\natural \epsilon_{\alpha\beta}\partial^\beta X^+ \,. \tag{4.27}$$

Then the first order nonlocal charge in the IIB background becomes $Q^{1*}_{[1]\rm IIB} = p^+ x^\natural$ where the zero mode of $X^\natural$ is $x^\natural$. The commutator between $Q^1_{[1]\rm IIB}$ and $Q^{1*}_{[1]\rm IIB}$ is realized as same as the one of $Q^1_{[0]\rm IIA}$ and $Q^{1*}_{[0]\rm IIA}$. The zero mode of $X^\natural$ is conjugate of w, $\left[x^\natural, wR/\alpha'\right] = i$ with $x^1_+ - x^1_- = x^\natural$ in the IIB background side. This WZ term (4.26) is turns out to be analogous to the Buscher T-duality transformation.

It is interesting that the M.C. equations for the one form current constructed from the IIA Noether currents (4.18) contain extra terms because of the IIA WZ term in (4.17). In order to compute the nonlocal currents in this case this anomaly must be treated consistently. We leave this problem for future investigation.

5. Conclusions and discussion

We have obtained nonlocal charges in terms of oscillators in a flat background and the IIB and IIA pp-wave backgrounds. For the flat background we have shown that the set of independent conserved nonlocal charges is the same before and after T-duality transformation with interchanging odd and even-order charges; for example the zero-th order charge in the T-dualized flat background coincides with the ones in the original background. T-duality interchanges the momenta and the winding number and the total Lorentz spin and the relative Lorentz spin which is the difference between the left mover's spin and the right

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mover's one. Among an infinite set of nonlocal charges independent charges are the zero-th and first order charges which are the momenta and the winding number of zero mode and the total spin and the relative spin for zero mode and non-zero modes separately.

For the pp-wave background we have computed the nonlocal charges and obtained expressions of the zero-th, first and second order ones in terms of oscillators. Contrast to the flat case coefficients of the mode expansion in nonlocal charges are different, so there exist an infinite set of independent nonlocal charges for the pp-wave case. We have shown that the zero-th order charges in the T-dualized pp-wave background, the IIA pp-wave background, are included as a part of an infinite set of nonlocal charges in the original IIB pp-wave background. Since we perform the lightcone quantization the lightcone Hamiltonians for the type IIB and the type IIA backgrounds are equal by T-duality. This equality leads to identification of the modes in both sides. As a result the zero-th order momentum charge in the IIA pp-wave side, $Q^1_{[0]\mathrm{IIA}}$, corresponds to the first order nonlocal momentum charge in the IIB pp-wave side, $Q^1_{[1]\mathrm{IIB}}$. In order to make this correspondence complete the zero-th order charge in the IIA pp-wave side, $Q^{1*}_{[0]\mathrm{IIA}}$, should correspond to the non-zero value of the first order charge $Q^{1*}_{[1]\mathrm{IIB}}$ in the IIB pp-wave side. Then we introduce a WZ term for the "IIB" pp-wave background in such a way that this term causes non-zero value of $Q^{1*}_{[1]\mathrm{IIB}}$ satisfying the corresponding algebra. It turns out that the Lagrange multiplier of the WZ term is a variable conjugate to the winding mode. This term is nothing but the term used in the Buscher T-duality transformation. In another word one can introduce the conjugate coordinate to the winding mode by adding the WZ term à la Buscher's T-duality transformation. The completeness of this correspondence requires the IIB side to add the $B_{\mu\nu}$ field as a target space interpretation of the WZ term and to include the relative coordinate $x^\natural = x_+ - x_-$, so these dual degrees of freedom are hidden in also the "IIB" side. Then it is natural to formulate string theories by "two-vierbein formalism" [32] and it may be generalize to the general field theories. It may be interesting to relate the issue to the finite size effect of the integrable system [33] and other properties [34].

In this paper we clarified the procedure of constructing the nonlocal charges for inhomogeneous SO(n) cosets such as a flat and the pp-wave background cases; the basic currents which satisfy the conservation law are set to be the Noether currents of the action, and the "flatness" condition is examined. Based on these currents nonlocal charges are constructed inductively. The conserved charges are obtained by the integration along the boundary of the semi-infinite strip cut open the cylinderical worldsheet. We could not compute the higher order nonlocal charges in the type IIA pp-wave background, since the "flatness" condition of the IIA Noether currents includes extra terms caused by the WZ term. We know that the WZ term produces a topological center in the Noether charge algebra, where charges can be constructed but the ground state is only invariant under a part of the symmetries. If this IIA theory and the IIB theory are really T-dual, then the infinite set of nonlocal charges also exist in both theories. So there may be a further generalization of the procedure constructing nonlocal charges for "non-flat" systems.

Nonlocal charges carry T-dual information as we have shown. T-duality in the pp-wave space may trace back to the one in the AdS space where an infinite set of nonlocal charges exist. So application of our analysis to the AdS space may be possible in the classical

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level. It is curious how nonlocal charges in the holographic dual theories realize T-dual information. Generalization involving U-duality may be interesting, and we leave these problem for future investigation.

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References

[1] D. Berenstein, J.M. Maldacena and H. Nastase, *Strings in flat space and pp waves from $N = 4$ super Yang-Mills*, *JHEP* **04** (2002) 013 [hep-th/0202021].

[2] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *A semi-classical limit of the gauge/string correspondence*, *Nucl. Phys.* **B 636** (2002) 99 [hep-th/0204051].

[3] S. Frolov and A.A. Tseytlin, *Semiclassical quantization of rotating superstring in $AdS_5 \times S^5$*, *JHEP* **06** (2002) 007 [hep-th/0204226].

[4] J.A. Minahan and K. Zarembo, *The Bethe-Ansatz for $N = 4$ super Yang-Mills*, *JHEP* **03** (2003) 013 [hep-th/0212208].

[5] S. Frolov and A.A. Tseytlin, *Rotating string solutions: AdS/CFT duality in non-supersymmetric sectors*, *Phys. Lett.* **B 570** (2003) 96 [hep-th/0306143].

[6] N. Beisert, C. Kristjansen and M. Staudacher, *The dilatation operator of $N = 4$ super Yang-Mills theory*, *Nucl. Phys.* **B 664** (2003) 131 [hep-th/0303060];
N. Beisert, *The complete one-loop dilatation operator of $N = 4$ super Yang-Mills theory*, *Nucl. Phys.* **B 676** (2004) 3 [hep-th/0307015].

[7] G. Arutyunov, S. Frolov, J. Russo and A.A. Tseytlin, *Spinning strings in $AdS_5 \times S^5$ and integrable systems*, *Nucl. Phys.* **B 671** (2003) 3 [hep-th/0307191];
G. Arutyunov, J. Russo and A.A. Tseytlin, *Spinning strings in $AdS_5 \times S^5$: new integrable system relations*, *Phys. Rev.* **D 69** (2004) 086009 [hep-th/0311004].

[8] N. Beisert and M. Staudacher, *The $N = 4$ SYM integrable super spin chain*, *Nucl. Phys.* **B 670** (2003) 439 [hep-th/0307042].

[9] N. Beisert, *The dilatation operator of $N = 4$ super Yang-Mills theory and integrability*, *Phys. Rept.* **405** (2005) 1 [hep-th/0407277];
K. Zarembo, *Semiclassical Bethe ansatz and AdS/CFT*, *Comptes Rendus Physique* **5** (2004) 1081–1090 [hep-th/0411191];
J. Plefka, *Spinning strings and integrable spin chains in the AdS/CFT correspondence*, hep-th/0507136.

[10] M. Lüscher, *Quantum nonlocal charges and absence of particle production in the two-dimensional nonlinear sigma model*, *Nucl. Phys.* **B 135** (1978) 1;
M. Lüscher and K. Pohlmeyer, *Scattering of massless lumps and nonlocal charges in the two-dimensional classical nonlinear sigma model*, *Nucl. Phys.* **B 137** (1978) 46.

[11] L.N. Lipatov, *High-energy asymptotics of multicolor QCD and exactly solvable lattice models*, hep-th/9311037;
L.D. Faddeev and G.P. Korchemsky, *High-energy QCD as a completely integrable model*, *Phys. Lett.* **B 342** (1995) 311 [hep-th/9404173].

[12] E. Brezin, C. Itzykson, J. Zinn-Justin and J.B. Zuber, *Remarks about the existence of nonlocal charges in two-dimensional models*, *Phys. Lett.* **B 82** (1979) 442.

[13] T.H. Buscher, *A symmetry of the string background field equations*, *Phys. Lett.* **B 194** (1987) 59; *Path integral derivation of quantum duality in nonlinear sigma models*, *Phys. Lett.* **B 201** (1988) 466.

[14] I. Bena, J. Polchinski and R. Roiban, *Hidden symmetries of the $AdS_5 \times S^5$ superstring*, *Phys. Rev.* **D 69** (2004) 046002 [hep-th/0305116].

[15] G. Mandal, N.V. Suryanarayana and S.R. Wadia, *Aspects of semiclassical strings in AdS_5*, *Phys. Lett.* **B 543** (2002) 81 [hep-th/0206103].

[16] M. Hatsuda and K. Yoshida, *Classical integrability and super yangian of superstring on $AdS_5 \times S^5$*, *Adv. Theor. Math. Phys.* **9** (2005) 703 [hep-th/0407044].

[17] B.C. Vallilo, *Flat currents in the classical $AdS_5 \times S^5$ pure spinor superstring*, *JHEP* **03** (2004) 037 [hep-th/0307018].

[18] N. Berkovits, *Brst cohomology and nonlocal conserved charges*, *JHEP* **02** (2005) 060 [hep-th/0409159]; *Quantum consistency of the superstring in $AdS_5 \times S^5$ background*, *JHEP* **03** (2005) 041 [hep-th/0411170].

[19] B.Y. Hou, D.T. Peng, C.H. Xiong and R.H. Yue, *The affine hidden symmetry and integrability of type-IIB superstring in $AdS_5 \times S^5$*, hep-th/0406239.

[20] M. Wolf, *On hidden symmetries of a super gauge theory and twistor string theory*, *JHEP* **02** (2005) 018 [hep-th/0412163].

[21] A. Das, J. Maharana, A. Melikyan and M. Sato, *The algebra of transition matrices for the $AdS_5 \times S^5$ superstring*, *JHEP* **12** (2004) 055 [hep-th/0411200].

[22] L.F. Alday, G. Arutyunov and A.A. Tseytlin, *On integrability of classical superstrings in $AdS_5 \times S^5$*, *JHEP* **07** (2005) 002 [hep-th/0502240].

[23] C.A.S. Young, *Non-local charges, $\mathbb{Z}_m$ gradings and coset space actions*, *Phys. Lett.* **B 632** (2006) 559 [hep-th/0503008].

[24] B.H. Miller, *Conserved charges in the principal chiral model on a supergroup*, hep-th/0602006.

[25] M. Hatsuda, *Sugawara form for AdS superstring*, *Nucl. Phys.* **B 730** (2005) 364 [hep-th/0507047].

[26] L.F. Alday, *Non-local charges on $AdS_5 \times S^5$ and pp-waves*, *JHEP* **12** (2003) 033 [hep-th/0310146].

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[27] J. Michelson, *(twisted) toroidal compactification of pp-waves*, *Phys. Rev.* **D 66** (2002) 066002 [hep-th/0203140].

[28] M. Alishahiha, M.A. Ganjali, A. Ghodsi and S. Parvizi, *On type-IIA string theory on the pp-wave background*, *Nucl. Phys.* **B 661** (2003) 174 [hep-th/0207037].

[29] S. Mizoguchi, T. Mogami and Y. Satoh, *A note on T-duality of strings in plane-wave backgrounds*, *Phys. Lett.* **B 564** (2003) 132 [hep-th/0302020].

[30] N. Berkovits, M. Bershadsky, T. Hauer, S. Zhukov and B. Zwiebach, *Superstring theory on $AdS_2 \times S^2$ as a coset supermanifold*, *Nucl. Phys.* **B 567** (2000) 61 [hep-th/9907200].

[31] S. Frolov, J. Plefka and M. Zamaklar, *The $AdS_5 \times S^5$ superstring in light-cone gauge and its bethe equations*, hep-th/0603008.

[32] W. Siegel, *Two vierbein formalism for string inspired axionic gravity*, *Phys. Rev.* **D 47** (1993) 5453 [hep-th/9302036].

[33] N. Mann and J. Polchinski, *Finite density states in integrable conformal field theories*, hep-th/0408162.

[34] T.L. Curtright and C.K. Zachos, *Nonlocal currents for supersymmetric nonlinear models*, *Phys. Rev.* **D 21** (1980) 411; *Supersymmetry and the nonlocal Yangian deformation symmetry*, *Nucl. Phys.* **B 402** (1993) 604 [hep-th/9210060]; *Currents, charges, and canonical structure of pseudodual chiral models*, *Phys. Rev.* **D 49** (1994) 5408 [hep-th/9401006];
B. Chen, Y.L. He, P. Zhang and X.C. Song, *Flat currents of the Green-Schwarz superstrings in $AdS_5 \times S^1$ and $AdS_3 \times S^3$ backgrounds*, *Phys. Rev.* **D 71** (2005) 086007 [hep-th/0503089];
E. Abdalla and A. Lima-Santos, *Classical non-local conserved charges in string theory*, hep-th/0506148.

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